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Hydromagnetic flow of fluid with variable viscosity in a uniform tube with peristalsis

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Abstract
We formulate this problem under an infinitely long wavelength approximation, a negligible Reynolds number and a small magnetic Reynolds number. We decide on a perturbation method of solution. The viscosity parameter $\alpha \ll 1$ is chosen as a perturbation parameter. The governing equations are developed up to first-order in the viscosity parameter (α). The zero-order system yields the classical Poiseuille flow when the Hartmann number M tends to zero. For the first-order system, we simplify a complicated group of products of Bessel functions by approximating the polynomial. The results show that the increasing magnetic field increases the pressure rise. In addition, the pressure rise increases as the viscosity parameter decreases at zero flow rate. Moreover, it is independent of the Hartmann number and viscosity parameter at certain values of flow rate. We make comparisons with other studies.

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1. Introduction

The application of magnets to the human body is called magnetotherapy, by which diseases are treated. No drugs are administered under this therapy. Magnets in the form of lodestones were known in ancient times and their properties were considered to be magical. They were, however, never considered as instruments of healing until the beginning of the sixteenth century when a Swiss alchemist and physician P A Paracelsus studied magnets and stated that they could heal inflammations, ulceration, and many diseases of the bowel (intestine) and uterus, and that they could also be useful both in internal as well as external ailments. It was his contention that any diseased part of the body, when exposed to a magnetic force, would be cured more effectively and more speedily than with a drug. Furthermore, Li *et al* (1994) have used an impulse magnetic field in the combined therapy of patients with stone fragments in the upper urinary tract. It was found that the impulse magnetic field (IMF) activates the impulse activity of the ureteral smooth muscles in 100% of cases. Furthermore, there is a non-invasive radiologic test that uses a magnetic field (not radiation) to evaluate organs in the abdomen prior to surgery in the small intestine (but not always). Moreover, in the ureter, the cells tend to be flattened towards the base and more rounded towards the epithelial surface. The attachments between cells are such that urine cannot pass between them. Also, although the epithelium is termed a 'mucosa' there are no mucus secreting cells or glands. We deduce that there is no absorption by the ureter wall, nor variable viscosity of the fluid passing in the ureter. In addition, Lykoudis and Roos (1970) considered that the viscosity in the ureter is constant. Inversely, all mucosal membranes produce mucus, in various parts of the gastrointestinal system (GI). So it is suitable to consider the small intestine application with hypothesis, magnetic field and fluid with variable viscosity. Due to the complexity of the non-linear equations of motion, we only consider the cases of an axisymmetric flow, an infinite wave train on the tube, an incompressible Newtonian fluid with variable viscosity and a constant transverse magnetic field. Expressions for the pressure rise per unit wavelength and friction force on the wall of tube (small intestine) have been computed numerically.

Many authors have studied the analysis of the mechanisms for peristaltic transport. Latham (1966) was probably the first to investigate the mechanism of peristalsis in relation to mechanical pumping. Lew *et al* (1971) suggested chyme as a non-Newtonian material having plastic-like properties. Shukla *et al* (1980) have investigated the effects of peripheral-layer viscosity on peristaltic transport of a bio-fluid in a uniform tube and have used the long wavelength approximation as in Shapiro *et al* (1969). Yin and Fung (1969) have investigated peristaltic waves in circular cylindrical tubes using a perturbation method. Bohme and Friedrich (1983) have investigated the peristaltic flow of viscoelastic liquids and have assumed that the relevant Reynolds number is small enough to neglect inertia forces, and that the ratio of the wavelength and the channel height is large, which implies that the pressure rise is constant over the cross section. Srivastava *et al* (1983) studied peristaltic transport of a fluid with variable viscosity through a non-uniform tube. Agrawal and Anwaruddin (1984) studied the effects of a magnetic field on blood flow by taking a simple mathematical model for blood through an equally branched channel with flexible outer walls executing peristaltic waves. Srivastava and Srivastava (1985) have investigated the effects of power-law fluid in uniform and non-uniform tubes under zero Reynolds number and long wavelength approximations. Pozrikidis (1987) has investigated a study of peristaltic flow under the assumption of creeping motion and has used the boundary integral method for Stokes flow. Siddiqui and Schwarz (1994) have investigated the peristaltic flow of a second-order fluid in a tube and have used a perturbation method to second order in dimensionless wavenumber. El Misery *et al* (1996) have investigated the peristaltic transport of Carreau fluid through a uniform channel, under zero Reynolds number and long wavelength approximations. El Shehawey *et al* (1998) have investigated the peristaltic transport of Carreau fluid through a non-uniform channel, under zero Reynolds number and long wavelength approximations. Abd El Hakeem and El-Misiery (2002) have investigated the peristaltic pumping of Carreau fluid in the presence of an endoscope. El-Misiery *et al* (2003) have studied the effects of an endoscope and fluid with variable viscosity on peristaltic motion.

With the above studies in mind, we propose to study the effects of a magnetic field and fluid with variable viscosity on peristaltic motion in a uniform tube.

2. Formulation and analysis

We consider the creeping flow of an incompressible Newtonian fluid with variable viscosity through an axisymmetric form in a uniform tube thickness with a sinusoidal wave travelling

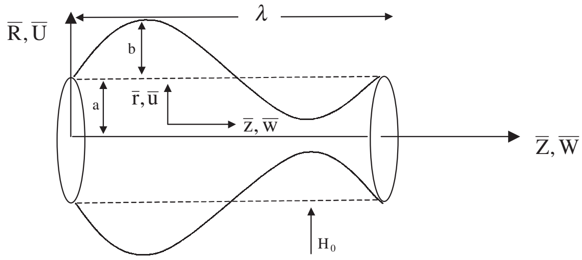

Figure 1. Peristaltic motion of fluid with variable viscosity subjected to a constant transverse magnetic.

down its wall. We assume that the fluid is subjected to a constant transverse magnetic field. The induced magnetic field is negligible, which is justified for flow at small magnetic Reynolds number. The external electric field is zero and the electric field due to polarization of charges is also negligible. Heat due to viscous and Joule dissipation is neglected. Also, we can neglect the gravity effect since the gravity transverse to the flow in the small intestine and it does not interact with fluid particles. The geometry of the wall surface is described in figure 1.

$$\bar{h} = a + b\sin\frac{2\pi}{\lambda}(\bar{Z} - c\bar{t}) \tag{2.1}$$

where a is the radius of the tube at inlet, b is the wave amplitude, λ is the wavelength, c is the wave speed and $\bar{t}$ is the time. We choose the cylindrical coordinate system $(\bar{R}, \bar{Z})$, where the $\bar{Z}$-axis lies along the centreline of the tube, and $\bar{R}$ transverse to it. In fixed coordinates $(\bar{R}, \bar{Z})$, the flow in the stationary coordinates is unsteady but if we choose moving coordinates $(\bar{r}, \bar{z})$, which travel in the $\bar{Z}$-direction with the same speed as the wave, then the flow can be treated as steady. The coordinate frames are related through

$$\bar{z} = \bar{Z} - c\bar{t} \qquad \bar{r} = \bar{R} \tag{2.2}$$

$$\bar{w} = \bar{W} - c \qquad \bar{u} = \bar{U} \tag{2.3}$$

where $\bar{U}$, $\bar{W}$ and $\bar{u}$, $\bar{w}$ are the velocity components in the radial and axial directions in the fixed and moving coordinates respectively.

The equations of motion and boundary conditions in the moving coordinates are the continuity equation

$$\frac{1}{\bar{r}}\frac{\partial(\bar{r}\bar{u})}{\partial\bar{r}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \tag{2.4}$$

and the Navier–Stokes equations

$$\frac{\partial\bar{P}}{\partial\bar{r}} = \frac{\partial}{\partial z}\left[\bar{\mu}(r)\left(\frac{\partial\bar{u}}{\partial\bar{z}} - \frac{\partial\bar{w}}{\partial\bar{r}}\right)\right] \tag{2.5}$$

$$\frac{\partial\bar{P}}{\partial\bar{z}} = -\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left[\bar{\mu}(r)\bar{r}\left(\frac{\partial\bar{u}}{\partial\bar{z}} - \frac{\partial\bar{w}}{\partial\bar{r}}\right)\right] - \sigma\mu_e^2 H_0^2\bar{w} \tag{2.6}$$

with the boundary conditions

$$\frac{\partial\bar{w}}{\partial\bar{r}} = 0 \qquad \bar{u} = 0 \qquad \text{at} \quad \bar{r} = 0 \tag{2.7a}$$

$$\bar{w} = -c \qquad \bar{u} = -c\frac{\mathrm{d}\bar{h}}{\mathrm{d}\bar{z}} \qquad \text{at} \quad \bar{r} = \bar{h} = a + b\sin\frac{2\pi}{\lambda}\bar{z} \tag{2.7b}$$

where $\bar{P}$ is the pressure, $\bar{\mu}(r)$ is the viscosity function, σ is the electrical conductivity and μ_e is the magnetic permeability.

It is convenient to non-dimensionalize the variables appearing in equations (2.1)–(2.7) introducing the wavenumber (δ) and the Hartmann number (M) as follows

$$R = \frac{\bar{R}}{a} \qquad r = \frac{\bar{r}}{a} \qquad Z = \frac{\bar{Z}}{\lambda} \qquad z = \frac{\bar{z}}{\lambda} \qquad W = \frac{\bar{W}}{c} \qquad w = \frac{\bar{w}}{c}$$

$$U = \frac{\lambda \bar{U}}{ac} \qquad \bar{u} = \frac{\lambda \bar{u}}{ac} \qquad P = \frac{a^2 \bar{P}}{c\lambda\mu_0} \qquad \mu = \frac{\bar{\mu}(r)}{\mu_0} \qquad t = \frac{c\bar{t}}{\lambda} \qquad \delta = \frac{a}{\lambda}$$

$$M = \mu_e H_0 a \sqrt{\frac{\sigma}{\mu_0}} \qquad h = \frac{\bar{h}}{a} = 1 + \phi \sin(2\pi z) \tag{2.8}$$

where ϕ is the amplitude ratio ($\phi = b/a$).

The equations of motion and boundary conditions become

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{2.9}$$

$$\frac{\partial P}{\partial r} = \delta^2 \mu(r) \frac{\partial}{\partial z}\left(\delta^2 \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}\right) \tag{2.10}$$

$$\frac{\partial P}{\partial z} = \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left[\mu(r) r \left(\frac{\partial w}{\partial r} - \delta^2 \frac{\partial u}{\partial z}\right)\right] - M^2 w \tag{2.11}$$

with the dimensionless boundary conditions

$$\frac{\partial w}{\partial r} = 0 \qquad u = 0 \quad \text{at} \quad r = 0 \tag{2.12a}$$

$$w = -1 \qquad u = -\frac{\mathrm{d}h}{\mathrm{d}z} \quad \text{at} \quad r = h = 1 + \phi \sin 2\pi z. \tag{2.12b}$$

Using the long wavelength approximation ($\delta = 0$) then equations (2.10) and (2.11) reduce to

$$\frac{\partial P}{\partial r} = 0 \tag{2.13}$$

$$\frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu(r)\frac{\partial w}{\partial r}\right) - M^2 w. \tag{2.14}$$

The effect of viscosity variation on peristaltic flow can be investigated for any given function $\mu(r)$. For the present investigation, we assume the viscosity variation in the dimensionless form as

$$\mu(r) = \mathrm{e}^{-\alpha r} \qquad \text{or} \qquad \mu(r) = 1 - \alpha r \qquad \text{for} \quad \alpha \ll 1 \tag{2.15}$$

where α is the viscosity parameter.

This assumption is reasonable for the following physiological reasons, since a normal person or animal of similar size consumes one to two litres of fluid every day. On top of that, another six to seven litres of fluid are received by the small intestine daily as secretions from salivary glands, stomach, pancreas, liver and the small intestine itself. Also, the viscosity of the gastric mucus (near the wall) varies between 1–10^{-2} cP but the viscosity of the chyme varies between 10^{-3}–10^{-6} cP, as reported in Shukla *et al* (1980). Moreover, viscosity depends on concentration through $\mu = \mathrm{e}^{\lambda C}$, where $\lambda \geqslant 1$ and C is the concentration. Thus, we can deduce that the concentration depends on radial distance. Hence, the viscosity of the fluid adjacent to the wall of the small intestine is less than that away from the wall. Therefore, the above choice of $\mu(r) = \mathrm{e}^{-\alpha r}$ is justified.

3. Rate of volume flow

The instantaneous volume flow rate in the fixed coordinate system is given by

$$\hat{Q} = 2\pi \int_0^{\bar{h}} \bar{R}\bar{W}\,\mathrm{d}\bar{R} \tag{3.1}$$

where $\bar{h}$ is a function of $\bar{Z}$ and $\bar{t}$. Substituting from equations (2.2) and (2.3) into equation (3.1), and then integrating yields

$$\hat{Q} = \bar{q} + \pi c\bar{h} \tag{3.2}$$

where

$$\bar{q} = 2\pi \int_0^{\bar{h}} \bar{r}\bar{w}\,\mathrm{d}\bar{r} \tag{3.3}$$

is the volume flow rate in the moving coordinates system and is independent of time. Here, $\bar{h}$ is a function of $\bar{z}$ alone and is defined through equation (2.8). Using the dimensionless variables, we find

$$F = \frac{\bar{q}}{2\pi a^2 c} = \int_0^h rw\,\mathrm{d}r. \tag{3.4}$$

The time-mean flow over a period $T = \frac{\lambda}{c}$ at a fixed Z-position is defined as

$$\bar{Q} = \frac{1}{T}\int_0^T \hat{Q}\,\mathrm{d}\bar{t}. \tag{3.5}$$

Substituting from equation (3.2) into equation (3.5) and integrating, we obtain

$$\bar{Q} = \bar{q} + \pi c\left(a^2 + \frac{b^2}{2}\right)$$

which may be written as

$$\frac{\bar{Q}}{2\pi ca^2} = \frac{\bar{q}}{2\pi ca^2} + \frac{1}{2}\left(1 + \frac{\phi^2}{2}\right). \tag{3.6}$$

On defining the dimensionless time-mean flow as

$$\Theta = \frac{\bar{Q}}{2\pi ca^2}$$

we rewrite equation (3.6) as

$$\Theta = F + \frac{1}{2}\left(1 + \frac{\phi^2}{2}\right). \tag{3.7}$$

4. Perturbation solution

We look for a regular perturbation in terms of small parameter α as follows

$$w = w_0 + \alpha w_1 + O(\alpha^2) \tag{4.1a}$$

$$u = u_0 + \alpha u_1 + O(\alpha^2) \tag{4.1b}$$

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \frac{\mathrm{d}P_0}{\mathrm{d}z} + \alpha\frac{\mathrm{d}P_1}{\mathrm{d}z} + O(\alpha^2) \tag{4.1c}$$

$$F = F_0 + \alpha F_1 + O(\alpha^2). \tag{4.1d}$$

Substituting from equations (4.1*a*)–(4.1*c*) in equations (2.9), (2.13), (2.14) and (3.4) we find: the system of order zero

$$\frac{1}{r}\frac{\partial(ru_0)}{\partial r}+\frac{\partial w_0}{\partial z}=0 \tag{4.2}$$

$$\frac{\partial P_0}{\partial r}=0 \tag{4.3}$$

$$\frac{\partial P_0}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_0}{\partial r}\right)-M^2 w_0 \tag{4.5}$$

with the dimensionless boundary conditions

$$\frac{\partial w_0}{\partial r}=0 \qquad u_0=0 \qquad \text{at} \quad r=0 \tag{4.6a}$$

$$w_0=-1 \qquad u_0=-\frac{\mathrm{d}h}{\mathrm{d}z} \qquad \text{at} \quad r=h=1+\phi\sin 2\pi z \tag{4.6b}$$

and the system of order one

$$\frac{1}{r}\frac{\partial(ru_1)}{\partial r}+\frac{\partial w_1}{\partial z}=0 \tag{4.7}$$

$$\frac{\partial P_1}{\partial r}=0 \tag{4.8}$$

$$\frac{\partial P_1}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(-r^2\frac{\partial w_0}{\partial r}+r\frac{\partial w_1}{\partial r}\right)-M^2 w_1 \tag{4.9}$$

with the dimensionless boundary conditions

$$\frac{\partial w_1}{\partial r}=0 \qquad u_1=0 \qquad \text{at} \quad r=0 \tag{4.10a}$$

$$w_1=0 \qquad u_1=0 \qquad \text{at} \quad r=h=1+\phi\sin 2\pi z. \tag{4.10b}$$

4.1. Solution of order zero

Solving equation (4.5) using equations (4.4) and (4.6) yields

$$w_0=\frac{\mathrm{d}P_0/\mathrm{d}z-M^2}{M^2 I_0(Mh)}[I_0(Mr)-I_0(Mh)]-1. \tag{4.11}$$

The volume flow rate F_0 in the moving coordinate system is given by

$$F_0=\int_0^h r w_0\,\mathrm{d}r. \tag{4.12}$$

Substituting from equation (4.11) into equation (4.12) and solving the result for $\frac{\mathrm{d}P_0}{\mathrm{d}z}$ yields

$$\frac{\mathrm{d}P_0}{\mathrm{d}z}=\frac{M^4 I_0(Mh)(2F_0+h^2)}{2MhI_1(Mh)-M^2h^2I_0(Mh)}+M^2. \tag{4.13}$$

4.2. Solution of order one

Substituting from equation (4.11) into equation (4.9) yields

$$r^2\frac{\partial^2 w_1}{\partial r^2}+r\frac{\partial w_1}{\partial r}-M^2r^2w_1=\frac{\mathrm{d}P_1}{\mathrm{d}z}r^2+g(z)[M^2r^3I_0(Mr)+Mr^2I_1(Mr)] \tag{4.14}$$

where

$$g(z)=\frac{\mathrm{d}P_0/\mathrm{d}z-M^2}{M^2I_0(Mh)}. \tag{4.15}$$

Differentiating equation (4.14) with respect to r yields

$$r^2\frac{\partial^2 S}{\partial r^2}+r\frac{\partial S}{\partial r}-(M^2r^2+1)S=g(z)[2M^2r^2I_0(Mr)+(M^3r^3-Mr)I_1(Mr)] \tag{4.16}$$

where

$$S=\frac{\partial w_1}{\partial r}. \tag{4.17}$$

The determination of a particular solution of equation (4.16) corresponding to this group of terms is complicated, and to avoid tedious manipulation we recall a similar group of terms. We represent the right-hand side of equation (4.16) by a polynomial in the following form

$$r^2\frac{\partial^2 S}{\partial r^2}+r\frac{\partial S}{\partial r}-(M^2r^2+1)S=g(z)\sum_{k=0}^{\infty}b_k(Mr)^{2k+2} \tag{4.18}$$

where

$$b_k=\frac{(2k+1)(2k+3)}{2^{2k+1}(\Gamma(k+1))^2(k+1)}\qquad \text{for}\quad k=0,1,2,3,\ldots. \tag{4.19}$$

The reason for the lower limit of the sum being zero and for the even power of r in the series is that, when the right-hand side of equation (4.16) is expanded in a power series in (Mr) using a series expansion of $I_0(Mr)$ and $I_1(Mr)$, we obtain only even power beginning with $(Mr)^2$ and then we can determine b_k.

Also, equation (4.14) can be written as

$$r^2\frac{\partial^2 w_1}{\partial r^2}+r\frac{\partial w_1}{\partial r}-M^2r^2w_1=\frac{\mathrm{d}P_1}{\mathrm{d}z}r^2+\frac{g(z)}{M}\sum_{k=0}^{\infty}\frac{b_k}{2k+1}(Mr)^{2k+2} \tag{4.20}$$

where b_k is defined through equation (4.19).

Solving equation (4.18) using equations (4.8) and (4.10) yields

$$w_1=\frac{\mathrm{d}P_1/\mathrm{d}z[I_0(Mr)-I_0(Mh)]}{M^2I_0(Mh)}+\frac{\mathrm{d}P_0/\mathrm{d}z-M^2}{M^3I_0(Mh)}\sum_{k=0}^{\infty}\frac{a_k(Mr)^{2k+3}}{2k+3}$$
$$-\frac{[\mathrm{d}P_0/\mathrm{d}z-M^2]I_0(Mr)}{M^3(I_0(Mh))^2}\sum_{k=0}^{\infty}\frac{a_k(Mh)^{2k+3}}{2k+3}. \tag{4.21}$$

The solution is obtained in equation (2.21) for an infinite series of the right-hand side of equation (4.20).

The volume flow rate F_1 in the moving coordinate system is given by

$$F_1=\int_0^h rw_1\,\mathrm{d}r. \tag{4.22}$$

Substituting from equation (4.21) into equation (4.22) and solving the result for $\frac{dP_1}{dz}$ yields

$$\frac{dP_1}{dz} = \frac{2F_1 M^4 I_0(Mh)}{2MhI_1(Mh) - (Mh)^2 I_0(Mh)} + A_1 \sum_{k=0}^{\infty} \frac{a_k (Mh)^{2k+3}}{2k+3} + A_2 \sum_{k=0}^{\infty} \frac{a_k (Mh)^{2k+5}}{2k+5} \qquad (4.23)$$

where $I_0(Mr)$ and $I_1(Mr)$ are the modified Bessel functions of the first kind and a_k, A_1 andA_2 are constants given by

$$a_0 = \frac{1}{2} \qquad a_k = \frac{b_k + a_{k-1}}{(2k+1)(2k+3)} \qquad \text{for} \quad k = 1, 2, 3, 4, \ldots \qquad (4.24)$$

$$A_1 = \frac{M^3(2F_0 + h^2)}{2MhI_1(Mh) - (Mh)^2 I_0(Mh)} \qquad A_2 = \frac{M^3 I_0(Mh)(2F_0 + h^2)}{[2MhI_1(Mh) - (Mh)^2 I_0(Mh)]^2}. \qquad (4.25)$$

Substituting from equations (4.11) and (4.21) into equation (4.1a) using the relation $\frac{dP_0}{dz} = \frac{dP}{dz} - \alpha \frac{dP_1}{dz}$ and neglecting terms greater than $O(\alpha)$, we obtain

$$w = \frac{(dP/dz - M^2)(I_0(Mr) - I_0(Mh))}{M^2 I_0(Mh)} + \alpha^2 \left\{ \frac{dP/dz - M^2}{M^3 I_0(Mh)} \sum_{k=0}^{\infty} \frac{a_k (Mr)^{2k+3}}{2k+3} - \frac{[dP/dz - M^2] I_0(Mr)}{M^3 (I_0(Mh))^2} \sum_{k=0}^{\infty} \frac{a_k (Mh)^{2k+3}}{2k+3} \right\}. \qquad (4.26)$$

Substituting from equations (4.13) and (4.23) into equation (4.1c) using the relation $F_0 = F - \alpha F_1$, where F is defined through equation (3.7), and neglecting the terms greater than $O(\alpha)$, we obtain

$$\frac{dP}{dz} = \frac{M^4 I_0(Mh)(2\Theta - \phi^2/2 - 1 + h^2)}{2MhI_1(Mh) - (Mh)^2 I_0(Mh)} + M^2 + \alpha \left\{ B_1 \sum_{k=0}^{\infty} \frac{a_k (Mh)^{2k+3}}{2k+3} + B_2 \sum_{k=0}^{\infty} \frac{a_k (Mh)^{2k+5}}{2k+5} \right\} \qquad (4.27)$$

where B_1 and B_2 are given by

$$B_1 = \frac{M^3(2\Theta - \phi^2/2 - 1 + h^2)}{2MhI_1(Mh) - (Mh)^2 I_0(Mh)} \qquad B_2 = \frac{M^3 I_0(Mh)(2\Theta - \phi^2/2 - 1 + h^2)}{[2MhI_1(Mh) - (Mh)^2 I_0(Mh)]^2}. \qquad (4.28)$$

The pressure rise ΔP_λ and friction force F_λ(on the wall) in the tube length λ in their non-dimensional forms are given by

$$\Delta P_\lambda = \int_0^1 \frac{dP}{dz} \, dz \qquad (4.29)$$

$$F_\lambda = \int_0^1 h^2 \left(-\frac{dP}{dz} \right) dz \qquad (4.30)$$

where dP/dz is defined through equation (4.27).

5. Numerical results and conclusions

We have used a regular perturbation series in terms of the dimensionless viscosity parameter (α) to obtain an analytical solution to the field equations for peristaltic flow of a Newtonian fluid in an axisymmetric tube. To study the behaviour of solutions, numerical calculations for several values of Hartmann number (M), viscosity parameter (α) and amplitude ratio (ϕ) have

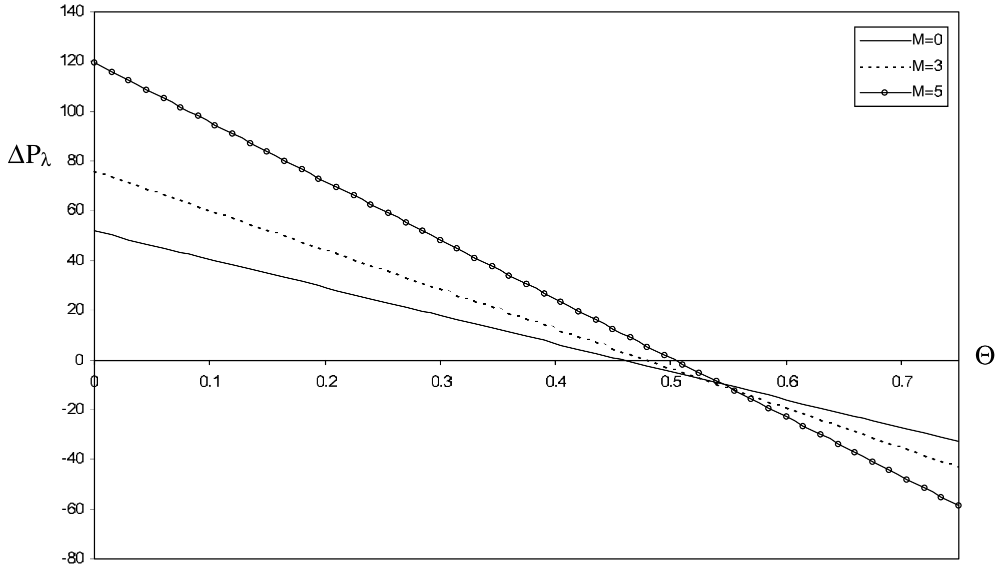

Figure 2. Pressure rise versus flow rate for $\phi = 0.6$ and $\alpha = 0.1$.

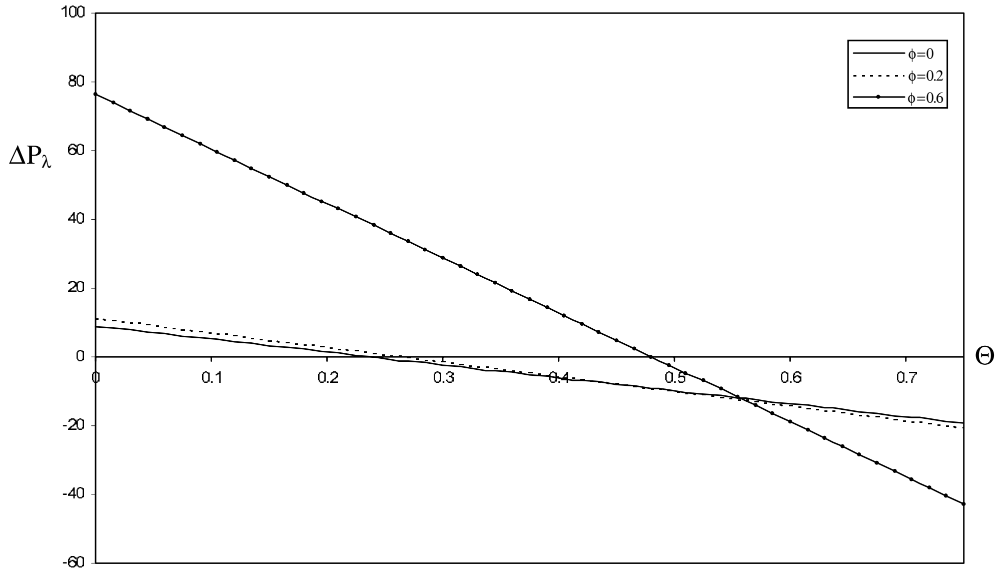

Figure 3. Pressure rise versus flow rate for $\alpha = 0.1$ and $M = 3$.

been carried out using a digital computer. Also, the infinity in equation (4.27) is approximated to 9 since the variation in pressure gradient $\mathrm{d}P/\mathrm{d}z$ is negligible at $k > 9$ for all values of the parameters of interest and all values of z. The relation between pressure rise and flow rate given by equation (4.29) is plotted in figures 2–4. The relation between friction force and flow rate given by equation (4.30) is plotted in figures 5–7. From a physiological point of view, there is no difference in peristaltic mechanism between the ureter and small intestine, as given

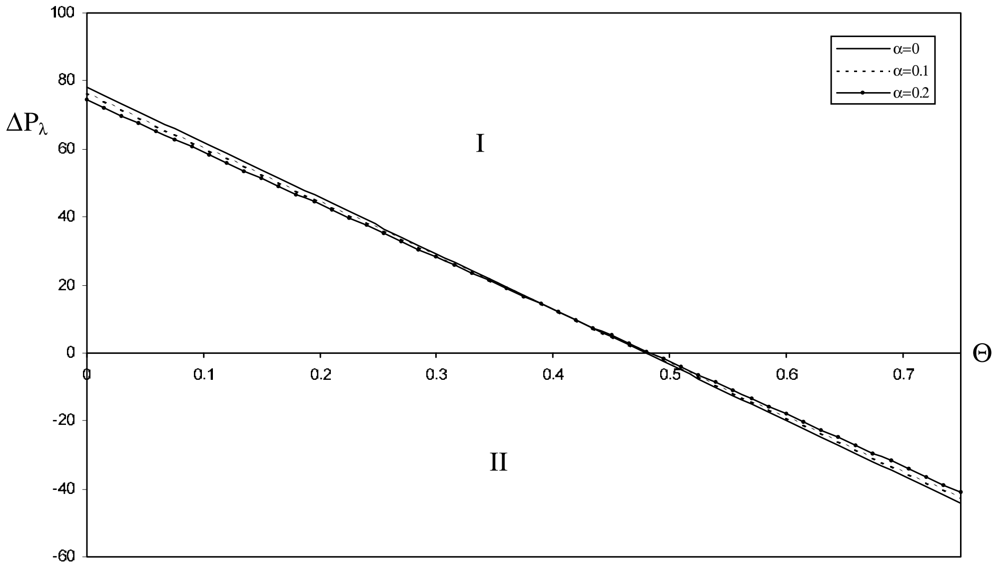

Figure 4. Pressure rise versus flow rate for $\phi = 0.6$. and $M = 3$.

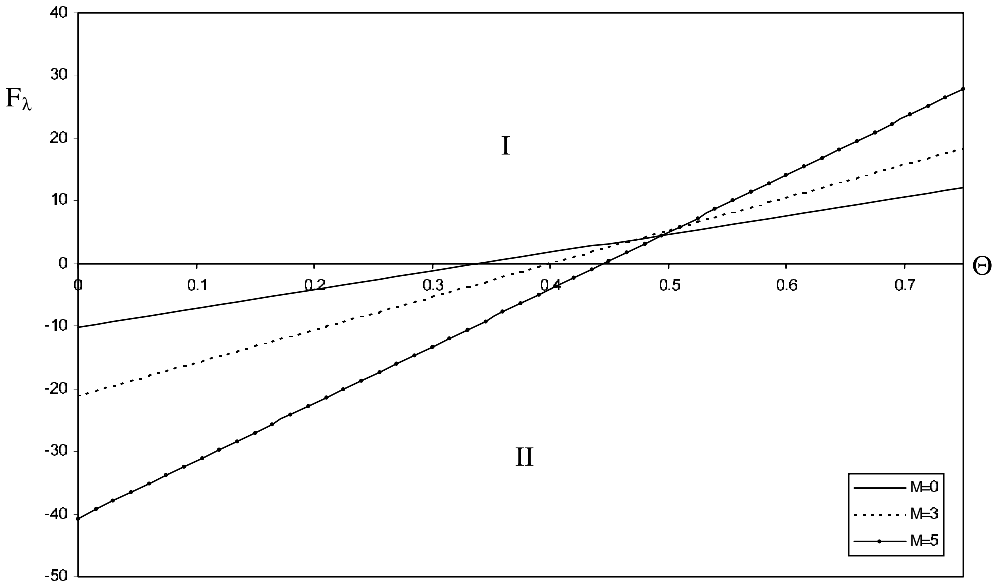

Figure 5. Friction force versus flow rate for $\phi = 0.6$ and $\alpha = 0.1$.

by Shukla *et al* (1980). The values of various parameters for physiological fluids in the small intestine, as reported in Shukla *et al* (1980) and Srivastava *et al* (1983), are

$$a = 1.25\ \text{cm} \qquad c = 2\ \text{cm}\,\text{min}^{-1} \qquad \lambda = 8.01\,\text{cm}.$$

It may be noted that the theory of long wavelength and zero Reynolds number of the present investigation remains applicable here, since the radius of the small intestine is very small compared with the wavelength.

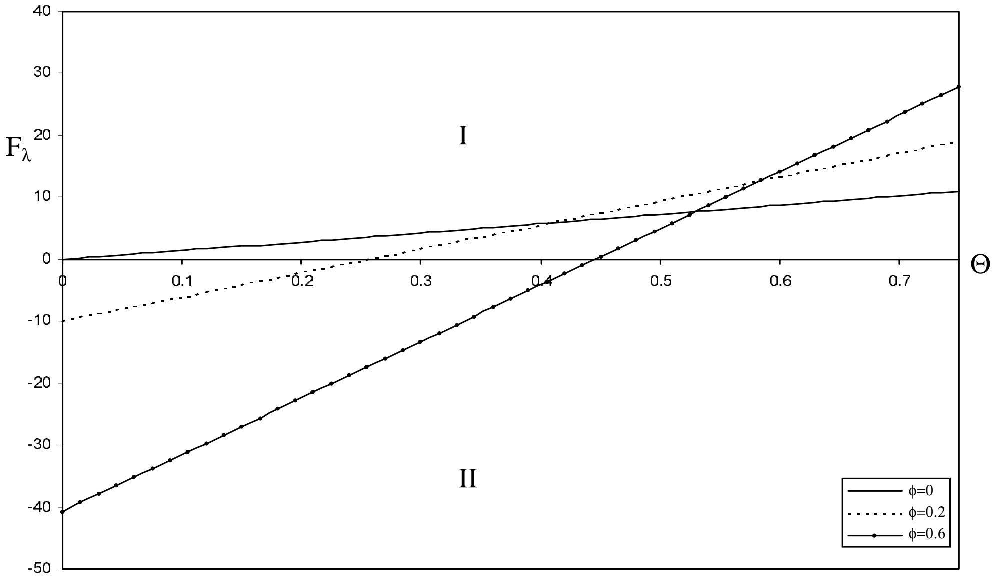

Figure 6. Friction force versus flow rate for $\alpha = 0.1$ and $M = 3$.

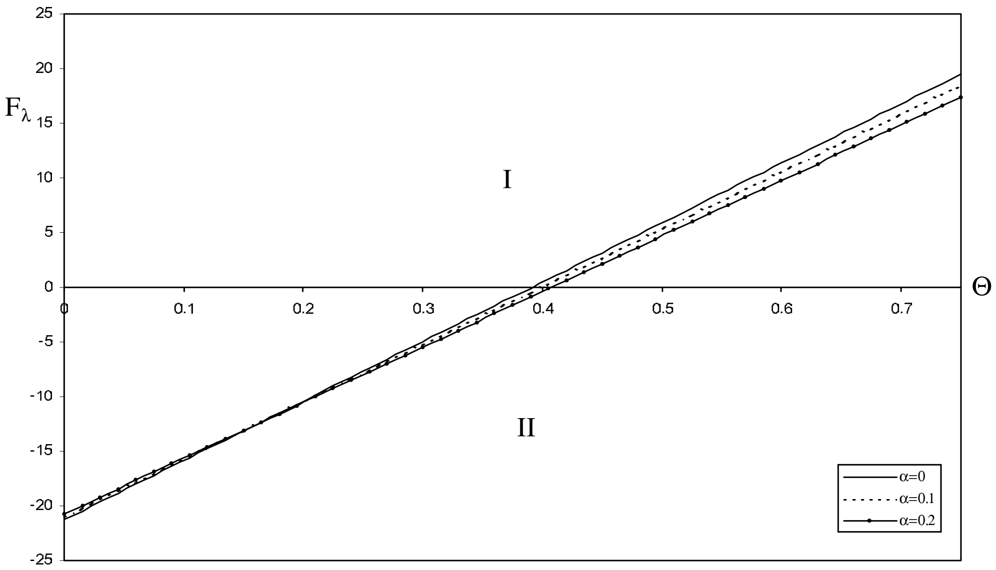

Figure 7. Friction force versus flow rate for $\phi = 0.6.$ and $M = 3$.

Figure 2 represents the variation of dimensionless pressure rise ΔP_λ with time-mean flow rate Θ at $\{\alpha = 0.1, \phi = 0.6, M = 0, 3, 5\}$ which shows a linear relation between them and maximum pressure rise occurs at zero flow rate for different values of Hartmann number. Also, pressure rise increases as flow rate decreases at $\{0 \leqslant \Theta < 0.45, M = 0\}$, $\{0 \leqslant \Theta < 0.48, M = 3\}$ and $\{0 \leqslant \Theta < 0.50, M = 5\}$ otherwise it increases with increasing flow rate. Furthermore, the pressure rise increases with increasing Hartmann number, and it is independent of Hartmann number variation at a certain value of flow rate.

Moreover, peristaltic pumping, where $\Theta > 0$ (positive pumping) and $\Delta P_\lambda > 0$ (adverse pressure gradient), occurs at $\{0 \leqslant \Theta < 0.45, M = 0\}$, $\{0 \leqslant \Theta < 0.48, M = 3\}$ and $\{0 \leqslant \Theta < 0.50, M = 5\}$ otherwise augmented pumping occurs, where $\Theta > 0$ (positive pumping) and $\Delta \mathrm{P}_\lambda < 0$ (favourable pressure gradient).

Figure 3 represents the variation of dimensionless pressure rise $\Delta \mathrm{P}_\lambda$ with time-mean flow rate Θ at $\{\alpha = 0.1, M = 3, \phi = 0$ (no peristalsis), 0.2 (small occlusion), 0.6 (high occlusion)$\}$. It is obvious that the pressure rise increases with increasing amplitude ratio and it is maximum at zero flow rate. Also, it is independent of flow amplitude ratio at certain values of flow rate. Furthermore, the peristaltic pumping occurs at $\{0 \leqslant \Theta < 0.23\ \phi = 0\}$, $\{0 \leqslant \Theta < 0.26, \phi = 0.2\}$ and $\{0 \leqslant \Theta < 0.48, \phi = 0.6\}$, otherwise augmented pumping occurs.

Figure 4 plots the relation between pressure rise and flow rate for different values of viscosity parameter (α) at $\phi = 0.6$. It is clear that an increase in flow rate decreases the pressure rise at $\{0 \leqslant \Theta < 0.48\}$ for different values of viscosity parameter, otherwise it increases with increasing flow rate. Also, the pressure rise increases with decreasing viscosity parameter and it is independent of viscosity parameter at a certain value of flow rate. Moreover, the peristaltic pumping occurs at $\{0 \leqslant \Theta < 0.48\}$, otherwise augmented pumping occurs.

In order to illustrate the effect of viscosity parameter and magnetic field on the friction force on the wall of the tube, figures 5–7 are sectored so that the upper region I denotes the region where the reflux phenomenon occurs, where F_λ is positive. Region II, where F_λ is negative, is designated as where peristaltic pumping occurs. In general, figures 2–7 show that the friction force has an opposite character in compression to the pressure rise.

The physical meaning of above discussion is considered as follows. Since the magnetic field stimulates the motor activity of the smooth muscles of the small intestine, then there is a contraction proximal to the bolus and relaxation distal. The contraction proximal causes a decrease in the radius of the small intestine and hence the amplitude ratio (ϕ) increases. In the same way, the relaxation distal causes an increase in the radius of the small intestine and decrease of the amplitude ratio. Thus, we conclude that the pressure rise increases at contraction proximal and decreases at relaxation distal. This agrees with our findings; namely, the pressure rise increases with increasing magnetic field and amplitude ratio. On the other hand, it decreases with decreasing magnetic field and amplitude ratio.

Comparing our results with other studies, we find that the velocity field obtained by Agrawal and Anwaruddin (1984) is a special case of our result, which is given by equation (4.26). Furthermore, if we put $k = 0$ (uniform channel) and using the transformations given by equations (2.2) and (2.3) in the results obtained by Srivastava and Srivastava (1983) then we obtain the same results when $M = 0$ in the present work. Furthermore, the results in the present work are more general than the experimental results obtained by Shapiro *et al* (1969). Moreover, comparing the results obtained through figure 3 with the results given by Siddiqui (1994), we find that for Newtonian fluid $\Delta P_\lambda < 0$ at $\phi = 0$ (no peristalsis) for all values of flow rate, but in the present work $\Delta P_\lambda > 0$ at $\{0 \leqslant \Theta < 0.225, \phi = 0\}$ and $\{0 \leqslant \Theta < 0.255, \phi = 0.2\}$ otherwise $\Delta P_\lambda < 0$. Also, our results without a magnetic field coincide with the results obtained by El Misery *et al* (2003) when there is no endoscope.

References

Abd El Hakeem Abd El Naby and El Misiery A E 2002 *J. Appl. Math. Comput.* **128** 19–35
Agrawal and Anwaruddin 1984 *J. Ranchi Uni. Math.* **15** 111–20
Barton C and Raynor S 1968 *J. Bull. Math. Biophys.* **30** 663–80
Bohme G and Friedrich R 1983 *J. Fluid Mech.* **128** 109–22
Bugliarllo G and Sevilla 1970 *J. Biorheology.* **7** 85–107
Burns J C and Parkes T 1967 *J. Fluid Mech.* **29** 731–43

El Misery A M, Abd El Hakeem Abd El Naby and Abd Hameed El Nagar 2003 *J. Phys. Soc. Japan* **72** 89–93

El Misery A M, Elshehawey E F and Hakeem A A 1996 *J. Phys. Soc. Japan* **65** 3524–9

Elshehawey E F, El Misery A M and Hakeem A A 1998 *J. Phys. Soc. Japan* **67** 434–40

Goldsmith H L and Skalak R 1975 *Ann. Rev. Inc. Palo Alto Publ.* **7** 231–47

Jaffrin M Y and Shapiro A H 1971 *Ann. Rev. Fluid Dyn.* **3** 13–37

Latham T W 1966 Fluid motion in a peristaltic pump *M S Thesis* MIT Cambridge MA

Lew H S, Fung Y C and Lowenstein C B J 1971 *Biomech.* **4** 297–315

Li A A, Nesterov N I, Malikova S N and Kiiatkin V A 1994 *Vopr Kurortol Fizioter Lech Fiz Kult (May–June 22–4)*

Lykoudis P S and Roos R 1970 *J. Fluid Mech.* **43** 661–74

Phukan K 2000 *Int. Conf. Forum for Interdisciplinary Mathematics, Hydromagnetic Flow and Heat Transfer over a Surface Stretching with a Power-Law Velocity Distribution (Indian Institute of Technology, Bombay, India (Dec. 19–21))*

Shapiro A H, Jaffrin M Y and Weinberg S L 1969 *J. Fluid Mech.* **37** 799–825

Shukla J B, Parihar R S, Rao R P and Gupta S P 1980 *J. Fluid Mech.* **97** 225–109

Siddiqui A M and Schwarz W H 1994 *Fluid Mech.* **53** 257–84

Srivastava L M and Srivastava V P 1985 *Ann. Biomed. Eng.* **13** 137–52

Srivastava L M, Srivastava V P and Sinha S N 1983 *Biorheol.* **20** 153–66

Yin F and Fung Y C 1969 *J. Appl. Mech.* **36** 579–87

Zien T F and Ostrach S A J 1970 *Biomech.* **3** 63–75